

SECTION: BLAST FROM THE PAST**UNITS:**

1. Significant Figures.
2. Decimal Places.
3. The Real Numbers.
4. Sets and Intervals.
5. Properties of Real Numbers.
6. Formula Rearrangement.
7. Simple applications of Percentage.
8. Glossary.

WHAT YOU NEED TO KNOW**1. Significant Figures**

- The first **s.f** is the **first non-zero digit** in the number, counting from the left.

e.x 1: 35123 e.x 2: 0.2365 e.x 3: 0.00178

- If we want a number to be rounded to a suitable degree of accuracy, for example to 2 s.f, we look the third s.f. If this is greater than or equal to 5 then round the second figure up. If rounding to 3 s.f, we look at the fourth s.f. and so on.

ex 1: Write 35123 correct to 4 s.f.

Ans: 5th s.f is 3. $3 < 5$ so 2 is not rounded up. Finally $35123 \rightarrow 35120$

ex 2: Write 0.2365 correct to 3 s.f.

Ans: 4th s.f is 5. $5 \geq 5$ so 6 rounds up to 7. Finally $0.2365 \rightarrow 0.237$

ex 3: Write 0.00178 correct to 2 s.f

Ans: 3th s.f is 8. $8 \geq 5$ so 7 rounds up to 8. Finally $0.00178 \rightarrow 0.0018$

2. Decimal Places

- The **decimal places** count after the decimal point (going from left to right).

- Rounding up or down follows the same rules as for s.f. So if the next number is greater than or equal to 5, then we round up.

ex 1: Write 5.12374 correct to 4 d.p.

Ans: 5th d.p is 4. $4 < 5$ so 7 is not rounded up.

Finally 5.12374 $\rightarrow$ 5.1237

ex 2: Write 0.1548 correct to 3 d.p.

Ans: 4th d.p is 8. $8 \geq 5$ so 4 rounds up to 5. Finally 0.1548 $\rightarrow$ 0.155

ex 3: Write 0.0078 correct to 2 d.p

Ans: 3th d.p is 7. $7 \geq 5$ so 0 rounds up to 1. Finally 0.0078 $\rightarrow$ 0.01

3. The Real Numbers

- A real number is any number that can be represented by a point on the real number line.



- Each point on the real number line corresponds to one and only real number and is called the coordinate of the point.
- The set of real numbers $\mathbb{R}$ contains some important subsets with which you should be familiar. Some of these are:

1. $\mathbb{N} = \{0, 1, 2, 3, \dots\}$, the set of **Natural** numbers.

2. $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$, the set of **Integers** (from the German word Zahl for number).

3. $\mathbb{Q} = \left\{ \frac{a}{b}, a, b \in \mathbb{Z}, b \neq 0 \right\}$, the set of **Rational** numbers.

4. $\mathbb{R} = \left\{ -1, 0.12, 0.3333\dots, 56, \frac{1}{2}, \sqrt{2}, \pi, \dots \right\}$, the set of **Real** numbers.

5. $\mathbb{R} - \mathbb{Q} =$

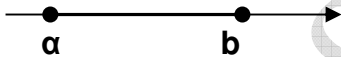
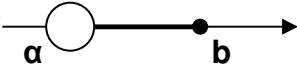
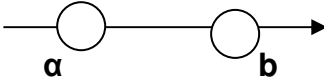
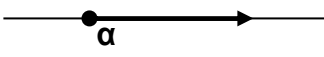
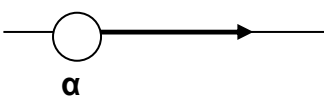
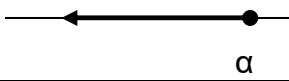
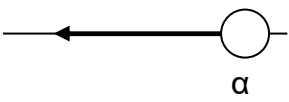
$\left\{ \text{A real number that cannot be written as the ratio of two integers, such that } \sqrt{2}, \pi, \dots \right\}$
 , the set of **Irrational** numbers.

- **Note that:** Every element of set N is also an element of set Z , every element of set Z is also an element of set Q , every element of set Q is also an element of set R and is written symbolically as $N \subseteq Z \subseteq Q \subseteq R$.
- Also, **we define:** $N^* = N - \{0\} = \{1, 2, 3, \dots\}$
 $Z^* = Z - \{0\} = \{\dots, -3, -2, -1, 1, 2, 3, \dots\}$ and $Z^+ = \{1, 2, 3, \dots\}$
 $R^* = R - \{0\}$.

4. Sets and Intervals

- A **set** is a collection of **distinct** objects.
- A set can be described by listing their elements within brackets (**ex.** $A = \{1, 2, 3\}$) or we can use a description in order to indicate which elements are included. **For example**, to indicate the set of positive integers x greater than 2, we could write: $A = \{x \in \mathbb{Z} / x > 2\}$.
- The symbol $\in$ indicates that a number or an element belongs to a set.
ex. $2 \in \mathbb{N}$, $-1 \in \mathbb{Z}$ and $\frac{1}{2} \in \mathbb{Q}$.
- The symbol $\notin$ indicates that a number is not an element of a set.
ex. $-2 \notin \mathbb{N}$, $\frac{1}{2} \notin \mathbb{Z}$.
- An empty (or null) set, is a set with no elements and can be written as $\emptyset$ or $\{\}$. **ex.** $A = \{x \in \mathbb{R} / x^2 = -1\}$.
- We show before that if every element of a set A is also an element of a set B , then we can write $A \subseteq B$ and we say that A is a **subset** of B . Equivalently, we can write $B \supseteq A$ and read as B is a **superset** of A .
ex. $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\} \Rightarrow A \subseteq B$ or $B \supseteq A$.
- If $A \subseteq B$ and A does not contain at least one element of the set B , we can write that $A \subset B$. Such a subset is called a **proper subset**.
ex. $A = \{1, 2, 3, \dots, 14\}$ and $B = \{1, 2, 3, \dots, 100\} \Rightarrow A \subseteq B \Rightarrow A \subset B$.

- A (real) **interval** is a set of real numbers with the property that any number that lies between two numbers in the set is also included in the set. **ex.** the set of all numbers x satisfying $0 \leq x \leq 1$ is an interval which contains 0 and 1, as well as all numbers between them.
- Some examples of intervals are:

	Inequality	Interval Notation	Geometrically Representation	Name
i)	$\alpha \leq x \leq b$	$x \in [\alpha, b]$		Closed and Bounded Interval (i.e both endpoints are included in the set)
ii)	$\alpha \leq x < b$	$x \in [\alpha, b[$		Half-Open and Bounded Interval (i.e includes one endpoint but not the other)
iii)	$\alpha < x \leq b$	$x \in]\alpha, b]$		Half-Open and Bounded Interval (i.e includes one endpoint but not the other)
iv)	$\alpha < x < b$	$x \in]\alpha, b[$		Open and Bounded Interval (i.e both endpoints are NOT included in the set)
v)	$x \geq \alpha$	$x \in [\alpha, +\infty[$		Half-Open and Unbounded Interval
vi)	$x > \alpha$	$x \in]\alpha, +\infty[$		Open and Unbounded Interval
vii)	$x \leq \alpha$	$x \in]-\infty, \alpha]$		Half-Open and Unbounded Interval
viii)	$x < \alpha$	$x \in]-\infty, \alpha[$		Open and Unbounded Interval

5. Properties of Real Numbers

- **Additive inverse** or **opposite** of a number α is $-\alpha$ since $\alpha + (-\alpha) = 0$.
- **Multiplicative inverse** or **reciprocal** of a number α is $\frac{1}{\alpha}$ since $\alpha \cdot \frac{1}{\alpha} = 1$. Remember that, the denominator of a fraction can never be 0, that's why zero has no reciprocal.
- **Even number** is an integer that can be divided exactly by 2 and every even number always end with a digit 0, 2, 4, 6 or 8. We can write an even number in the form of $2 \cdot n$, $n \in \mathbb{Z}$.
- **Odd** is an integer that is not an even and always end with a digit 1, 3, 5, 7 or 9. We can write an odd number in the form of $2 \cdot n + 1$, $n \in \mathbb{Z}$.
- **Prime numbers** are the numbers which are divided only by 1 and themselves (excluding 1). **ex.** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ...
- **Absolute value** or **modulus** of a number α , denoted by $|a|$, is the distance from a to 0 on the real number line. Since the distance must be positive or zero, we define that: $|a| = \begin{cases} \alpha, & \text{if } \alpha \geq 0 \\ -\alpha & \text{if } \alpha < 0 \end{cases}$.
- Given that α and b are real numbers, the distance between the points with coordinates α and b on the real number line is: $|b - a| = |\alpha - b|$.
- **PROPERTIES OF ADDITION**
 - (i) **Commutative Property:** $\alpha + b = b + \alpha$.
 - (ii) **Associative Property:** $\alpha + (b + c) = (\alpha + b) + c$.
 - (iii) **Additive Identity Property:** $\alpha + 0 = \alpha$.
 - (iv) **Additive Inverse Property:** $\alpha + (-\alpha) = 0$.

Remember that:

To **add fractions**,

1. Make sure that the bottom numbers (denominators) are the same.
2. Then add the top numbers (the numerators) and put the answer over the denominator.
3. Finally, simplify if needed.

ex1. $\frac{\alpha}{c} \pm \frac{b}{c} = \frac{\alpha \pm b}{c}, c \neq 0$.

ex2. $\frac{\alpha}{b} \pm \frac{c}{d} = \frac{\alpha \cdot d}{bd} \pm \frac{c \cdot b}{bd} = \frac{\alpha d \pm cb}{bd}, b, d \neq 0.$

• **PROPERTIES OF MULTIPLICATION**

- (i) **Commutative Property:** $\alpha \cdot b = b \cdot \alpha.$
- (ii) **Associative Property:** $\alpha \cdot (b \cdot c) = (\alpha \cdot b) \cdot c.$
- (iii) **Multiplicative Identity Property:** $\alpha \cdot 1 = \alpha.$
- (iv) **Multiplicative Inverse Property:** $\alpha \cdot \frac{1}{\alpha} = 1, \alpha \neq 0.$
- (v) $\alpha \cdot 0 = 0$ and $\frac{0}{\alpha} = 0, \alpha \neq 0.$

Remember that:

To **multiply fractions**,

1. Simplify the fractions if not in lowest terms.
2. Then multiply the numerators of the fractions to get the new numerator.
3. Multiply the denominators of the fractions to get the new denominators.

ex1. $\frac{\alpha}{b} \cdot \frac{c}{d} = \frac{\alpha \cdot c}{b \cdot d}, b, d \neq 0.$

ex2. $\alpha \cdot \frac{b}{c} = \frac{\alpha}{1} \cdot \frac{b}{c} = \frac{\alpha \cdot b}{1 \cdot c} = \frac{\alpha b}{c}, c \neq 0.$

But, in order to divide two fractions, we keep the first fraction and we turn the division sign into a multiplication sign flipping the second fraction over (find its reciprocal).

ex3. $\frac{\alpha}{b} : \frac{c}{d} = \frac{\alpha}{b} \cdot \frac{d}{c} = \frac{\alpha d}{bc}, c, b, d \neq 0$

ex4. $\frac{\frac{\alpha}{b}}{\frac{c}{d}} = \frac{\alpha d}{bc}, c, b, d \neq 0$ (**Compound Fraction**).

• **Notice that:**

Multiplication	Division
$\alpha \cdot 0 = 0$	$\alpha : 0 = \text{Not Defined}$
$\alpha \cdot 1 = \alpha$	$0 : \alpha = 0$
$\alpha \cdot \alpha = \alpha^2$	$\alpha : 1 = \alpha$
But $\alpha + \alpha = 2\alpha$	$\alpha : \alpha = 1$

- DISTRIBUTIVE PROPERTY**

- (i) $\alpha(b \pm c) = \alpha b \pm \alpha c$.
- (ii) $(\alpha + b)(c + d) = \alpha c + \alpha d + bc + bd$.

- REMEMBER THAT**

(i) $\alpha \cdot b = 0 \Leftrightarrow \alpha = 0 \text{ or } b = 0$

(ii) $\alpha \cdot b \neq 0 \Leftrightarrow \alpha \neq 0 \text{ and } b \neq 0$

- EQUIVALENT FRACTIONS PROPERTIES**

- (i) $\frac{\alpha}{b} = \frac{c}{d} \Leftrightarrow \alpha \cdot d = b \cdot c, b \cdot d \neq 0$
- (ii) $\frac{\alpha}{b} = \frac{c}{d} \Leftrightarrow \frac{\alpha}{c} = \frac{b}{d}, bcd \neq 0$
- (iii) $\frac{\alpha}{b} = \frac{c}{d} \Leftrightarrow \frac{\alpha + b}{b} = \frac{c + d}{d}, bd \neq 0$
- (iv) $\frac{\alpha}{b} = \frac{c}{d}$ then $\frac{\alpha}{b} = \frac{c}{d} = \frac{\alpha + c}{b + d}, bd(b + d) \neq 0$

- EXPONENTIAL PROPERTIES**

1. $a^1 = a$.
2. $a^0 = 1, a \neq 0$.
3. $a^{-k} = \frac{1}{a^k}, a \neq 0$.
4. $a^k \cdot a^w = a^{k+w}$.
5. $\frac{a^k}{a^w} = a^{k-w}, a \neq 0$.
6. $(a \cdot b)^k = a^k \cdot b^k$.
7. $\left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}$.
8. $(a^k)^w = a^{k \cdot w}$.
9. If **k** is an **odd** number, then $a^k = b^k \Leftrightarrow a = b$.
ex. $x^3 = 8 \Leftrightarrow x^3 = 2^3 \Leftrightarrow x = 2$.
10. If **k** is an **even** number, then $a^k = b^k \Leftrightarrow a = \pm b$.
ex. $x^2 = 4 \Leftrightarrow x^2 = 2^2 \Leftrightarrow x = 2 \text{ or } x = -2$.

11. If $a \neq 1$, then $a^k = a^w \Leftrightarrow k = w$.

ex. $2^x = 8 \Leftrightarrow 2^x = 2^3 \Leftrightarrow x = 3$.

• **Remember that:**

$$10^1 = 10$$

$$10^{-1} = 0,1$$

$$10^2 = 100$$

$$10^{-2} = 0,01$$

1. $10^3 = 1000$ **and** $10^{-3} = 0,001$

.....

.....

$$10^n = \underbrace{100 \dots 000}_{n \text{ zeros}}$$

$$10^{-n} = \underbrace{0,00 \dots 01}_{n \text{ zeros}}$$

2. **Standard form:** $a \cdot 10^k$ with $1 \leq a < 10, k \in \mathbb{Z}$.

ex1. $-2500000000 = -2,5 \cdot 10^9$

ex2. $0,00000000035 = 3,5 \cdot 10^{-10}$

ex3. $6977.55832 = 6.97755832 \cdot 10^3 = 6.98 \cdot 10^3$ at 3s.f.

• **BASIC IDENTITIES**

1. $(a+b)^2 = a^2 + 2ab + b^2$.

2. $(a-b)^2 = a^2 - 2ab + b^2$.

3. $a^2 - b^2 = (a-b) \cdot (a+b)$.

4. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$.

5. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

6. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

7. $a^3 + b^3 = (a+b) \cdot (a^2 - ab + b^2)$.

8. $a^3 - b^3 = (a-b) \cdot (a^2 + ab + b^2)$.

• **BASIC FACTORIZATION METHODS**

1. Common factor: $\alpha b + \alpha c = \alpha(b+c)$.

2. Grouping: $\alpha x + \alpha y + bx + by = \alpha(x+y) + b(x+y) = (x+y)(\alpha+b)$.

3. Difference in two squares: $\alpha^2 - b^2 = (\alpha+b)(\alpha-b)$.

4. Sum or Difference of two cubes:
$$\begin{cases} \alpha^3 - b^3 = (\alpha-b)(\alpha^2 + \alpha b + b^2) \\ \alpha^3 + b^3 = (\alpha+b)(\alpha^2 - \alpha b + b^2) \end{cases}$$

5. Perfect squares Trinomials:
$$\begin{cases} \alpha^2 + 2\alpha b + b^2 = (\alpha+b)^2 \\ \alpha^2 - 2\alpha b + b^2 = (\alpha-b)^2 \end{cases}$$

6. Trinomials: $x^2 + (\alpha+b)x + \alpha b = (x+\alpha)(x+b)$.

- **INEQUALITY PROPERTIES**

1. $a > b \Leftrightarrow a - b > 0$.
2. $a < b \Leftrightarrow a - b < 0$.
3. $a^2 \geq 0$ for every $a \in \mathbb{R}$.
4. If $a > b$ and $b > c$ then $a > c$.
5. $a > b \Leftrightarrow a \pm c > b \pm c$.
6. If $c > 0$, then $a > b \Leftrightarrow ac > bc$ and $a > b \Leftrightarrow \frac{a}{c} > \frac{b}{c}$.
7. If $c < 0$, then $a > b \Leftrightarrow ac < bc$ and $a > b \Leftrightarrow \frac{a}{c} < \frac{b}{c}$.
8. If a, b two numbers with the same sign, then $a > b \Leftrightarrow \frac{1}{a} < \frac{1}{b}$.
9. If a, b positive numbers, then $a > b \Leftrightarrow a^k > b^k, k \in \mathbb{N}^*$.
10. If $\left. \begin{matrix} a > b \\ c > d \end{matrix} \right\} \Rightarrow a + c > b + d$
11. If a, b, c, d positive numbers, then $\left. \begin{matrix} a > b \\ c > d \end{matrix} \right\} \Rightarrow ac > bd$

 **Notice that:** We never subtract or divide inequalities.

- **ABSOLUTE VALUE PROPERTIES**

1. $|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$.
2. $|a| \geq 0$ for every $a \in \mathbb{R}$.
3. $|a| = |-a|$.
4. $-|a| \leq a \leq |a|$ for every $a \in \mathbb{R}$.
5. $|a| + |b| = 0 \Leftrightarrow a = b = 0$.
6. $|x| = a > 0 \Leftrightarrow x = a \text{ or } x = -a$.
7. $|x| = |y| \Leftrightarrow x = \pm y$.
8. $|x \cdot y| = |x| \cdot |y|$.
9. $\left| \frac{x}{y} \right| = \frac{|x|}{|y|}$.
10. $|x + y| \leq |x| + |y|$.
11. $|x - y| \leq |x| + |y|$.
12. $|x| \leq k \Leftrightarrow -k \leq x \leq k$.
13. $|x| \geq k \Leftrightarrow x > k \text{ or } x < -k$.

- **ROOTS AND RADICALS PROPERTIES**

1. $\sqrt[k]{a} = b \Leftrightarrow a = b^k, a, b \geq 0, k \in \mathbb{N}^*.$
2. $\sqrt[k]{a} \cdot \sqrt[k]{b} = \sqrt[k]{ab}, a, b \geq 0, k \in \mathbb{N}^*.$
3. $\frac{\sqrt[k]{a}}{\sqrt[k]{b}} = \sqrt[k]{\frac{a}{b}}, a \geq 0, b > 0.$
4. $\sqrt[k]{a} \geq \sqrt[k]{b} \Leftrightarrow a \geq b.$
5. If k an even number, then $\sqrt[k]{a^k} = |a|, a \in \mathbb{R}.$
6. If k an even number, then $\sqrt[k]{a^k} = a, a \geq 0.$
7. $a^{\frac{k}{n}} = \sqrt[n]{a^k}, a > 0, n, k \in \mathbb{N}^*.$

6. Formula Rearrangement

- To **rearrange** a formula, we use the same processes as used for solving an equation for the variable we wish to be the subject.

ex. Make **z** the subject of $a + bz = \frac{y}{2}.$

Ans:

$$a + bz = \frac{y}{2} \text{ (multiply both sides by 2)}$$

$$2a + 2bz = 2 \cdot \frac{y}{2}$$

$$2a + 2bz = y \text{ (solve for } 2bz)$$

$$2bz = y - 2a \text{ (divide both sides by } 2b)$$

$$z = \frac{y - 2a}{2b}$$

7. Simple applications of Ratio, Percentage and Proportion

- **Percentage** is a number or ratio expressed as a fraction of 100 and is often denoted using the percent sign %. **i.e** $a\% = \frac{a}{100}.$

- The percentage **a% of b** is equal to: $a\% \cdot b = \frac{a}{100} \cdot b.$

ex. Find the 5% of 30€. **Ans:** $5\% \cdot 30\text{€} = \frac{5}{100} \cdot 30 = 1.5\text{€}.$

- The % α mark of a quantity b out of c , is equal to: $\frac{b}{c} \cdot 100\% = a\%$.

ex. Find in % mark of 15 out of 25.

Ans: $\frac{15}{25} \cdot 100\% = \frac{3}{5} \cdot 100\% = 0.6 \cdot 100\% = 60\%$.

- The **percentage change**:

- The **percentage increase** is equal to:

$$\alpha\% = \frac{\text{New Value} - \text{Original Value}}{\text{Original Value}} \cdot 100\% = \frac{\text{Change in Value}}{\text{Original Value}} \cdot 100\%.$$

ex. The salary of an employee was 7200€ and after his promotion he is going to get 7776€. Find the percentage of increase.

Ans:

$$\begin{aligned} \alpha\% &= \frac{\text{New number} - \text{Original number}}{\text{Original number}} \cdot 100\% = \frac{7776 - 7200}{7200} \cdot 100\% = \\ &= \frac{576}{7200} \cdot 100\% = 0,08 \cdot 100\% = 8\% \end{aligned}$$

- The **percentage decrease** is equal to:

$$\alpha\% = \frac{\text{Original Value} - \text{New Value}}{\text{Original Value}} \cdot 100\% = \frac{\text{Change in Value}}{\text{Original Value}} \cdot 100\%.$$

ex. The original value of a car was 20000€. After sales we can buy the same car at a value of 17000€. Find the percentage of decrease.

Ans:

$$\begin{aligned} \alpha\% &= \frac{\text{Original number} - \text{New number}}{\text{Original number}} \cdot 100\% = \frac{20000 - 17000}{20000} \cdot 100\% = \\ &= \frac{3000}{20000} \cdot 100\% = 0,15 \cdot 100\% = 15\% \end{aligned}$$

3. The **percentage error** measures how accurate an experiment is, compared to the expected numerical outcome.

$$\% \text{ error} = \frac{\text{Experimental Value} - \text{Theoretical Value}}{\text{Theoretical Value}} \cdot 100\% = \frac{\text{Change in Value}}{\text{Theoretical Value}} \cdot 100\%$$

ex. Demar estimated that it would take him 5 hours to drive to Thessaloniki from Athens, but when he made the trip, it took him 4 hours. Find the % error in his estimation.

Ans:

$$\begin{aligned} \% \text{ error} &= \frac{\text{Actual Value} - \text{Expected Value}}{\text{Expected Value}} \cdot 100\% = \\ &= \frac{\text{Change in Value}}{\text{Expected Value}} \cdot 100\% = \frac{4h - 5h}{5h} \cdot 100\% = \\ &= \frac{-1}{5h} \cdot 100\% = -0,2 \cdot 100\% = -20\% \end{aligned}$$

8. Command Terms Glossary

- You should be familiar with the command terms in your Diploma Programme subjects as they are used frequently in examination questions and the course content. The command terms give you clear direction regarding what is expected from you in each exam question.

COMMAND TERM	DEFINITION
Annotate	Add brief notes to a diagram or graph.
Apply	Use an idea, equation, principle, theory or law in relation to a given problem or issue. (See also "Use")
Calculate	Obtain a numerical answer showing the relevant stages in the working.
Comment	Give a judgment based on a given statement or result of a calculation.
Construct	Display information in a diagrammatic or logical form.
Contrast	Give an account of the differences between two (or more) items or situations, referring to both (all) of them throughout.
Deduce	Show a result using known information.
Demonstrate	Make clear by reasoning or evidence, illustrating with examples or practical application.
Derive	Manipulate a mathematical relationship to give a new equation or relationship.

Describe	Give a detailed account.
Design	Produce a plan, simulation or model.
Determine	Obtain the only possible answer.
Differentiate	Obtain the derivative of a function.
Discuss	Offer a considered and balanced review that includes a range of arguments, factors or hypotheses. Opinions or conclusions should be presented clearly and supported by appropriate evidence.
Distinguish	Make clear the differences between two or more concepts or items.
Draw	Represent by means of a labelled, accurate diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Estimate	Obtain an approximate value.
Evaluate	Make an appraisal by weighing up the strengths and limitations.
Explain	Give a detailed account, including reasons or causes. (See also “Justify”.)
Find	Obtain an answer showing relevant stages in the working.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.
Identify	Provide an answer from a number of possibilities.
Integrate	Obtain the integral of a function.
Interpret	Use knowledge and understanding to recognize trends and draw conclusions from given information.
Investigate	Observe, study, or make a detailed and systematic examination, in order to establish facts and reach new conclusions.
Justify	Give valid reasons or evidence to support an answer or conclusion. (See also “Explain”.)
Label	Add labels to a diagram.
List	Give a sequence of brief answers with no explanation.
Measure	Obtain a value for a quantity.
Plot	Mark the position of points on a diagram.
Predict	Give an expected result.
Prove	Use a sequence of logical steps to obtain the required result in a formal way.
Show	Give the steps in a calculation or derivation.

Show that	Obtain the required result (possibly using information given) without the formality of proof. "Show that" questions do not generally require the use of a calculator.
Sketch	Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Solve	Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.
State	Give a specific name, value or other brief answer without explanation or calculation.
Suggest	Propose a solution, hypothesis or other possible answer.
Verify	Provide evidence that validates the result.
Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.

WORKED EXAMPLES

1. Simplify the following:

$$(i) \quad (-3x^3y)^2 \qquad (ii) \quad 2x^2y \cdot \left(-\frac{3}{4}x^3y^2\right)$$

$$(iii) \quad \left(-\frac{2}{3}x^2\right)^3 \cdot (-x^3)^2 \qquad (iv) \quad (-x^2y)^3 \cdot (-2xy)^2 : (x^8y^7)$$

Solution

$$(i) \quad (-3x^3y)^2 = (-3)^2 \cdot (x^3)^2 \cdot y^2 = 9x^{2 \cdot 3}y^2 = 9x^6y^2.$$

$$(ii) \quad 2x^2y \cdot \left(-\frac{3}{4}x^3y^2\right) = 2 \cdot \left(-\frac{3}{4}\right) \cdot x^2x^3 \cdot y \cdot y^2 = -\frac{6}{4} \cdot x^{2+3} \cdot y^{1+2} = -\frac{3}{2} \cdot x^5 \cdot y^3.$$

$$(iii) \quad \left(-\frac{2}{3}x^2\right)^3 \cdot (-x^3)^2 = \left(-\frac{2}{3}\right)^3 \cdot (x^2)^3 \cdot (x^3)^2 = -\frac{2^3}{3^3} \cdot x^6 \cdot x^6 = -\frac{8}{27} \cdot x^{12}.$$

$$(iv) \quad (-x^2y)^3 \cdot (-2xy)^2 : (x^8y^7) = \frac{(-x^2y)^3 \cdot (-2xy)^2}{(x^8y^7)} = \frac{-x^6y^3 \cdot 4x^2y^2}{x^8y^7} = -\frac{4x^8y^5}{x^8y^7} = -4y^{5-7} = -4y^{-2}$$

2. Evaluate the following exponential expressions:

(i) $A = (-15)^0 - (-5)^2 - 5^2 - 4^3 - (-4)^3.$

(ii) $B = (-2)^x - \left(-\frac{2}{3}\right)^{x-1} - \left(-\frac{1}{2}\right)^{x-2}, \text{ for } x = -1.$

Solution

(i) $A = (-15)^0 - (-5)^2 - 5^2 - 4^3 - (-4)^3 = 1 - 5^2 - 25 - 64 + 4^3 = 1 - 25 - 25 - 64 + 64 = -49$

(ii)

$$\begin{aligned} B &= (-2)^x - \left(-\frac{2}{3}\right)^{x-1} - \left(-\frac{1}{2}\right)^{x-2} = (-2)^{-1} - \left(-\frac{2}{3}\right)^{-1-1} - \left(-\frac{1}{2}\right)^{-1-2} = -2^{-1} - \left(-\frac{2}{3}\right)^{-2} - \left(-\frac{1}{2}\right)^{-3} \\ &= -\frac{1}{2} - \left(\frac{2}{3}\right)^{-2} + \left(\frac{1}{2}\right)^{-3} = -\frac{1}{2} - \left(\frac{3}{2}\right)^2 + (2)^3 = -\frac{1}{2} - \frac{9}{4} + 8 = \frac{21}{4} \end{aligned}$$

3. Prove that:

(i) $\sqrt{12} = 2\sqrt{3}$

(ii) $\sqrt{6} \cdot \sqrt{14} = 2\sqrt{21}$

(iii) $\sqrt{12} - \sqrt{75} = -3\sqrt{3}.$

Solution

(i) $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}.$

(ii) $\sqrt{6} \cdot \sqrt{14} = \sqrt{6 \cdot 14} = \sqrt{84} = \sqrt{4 \cdot 21} = \sqrt{4} \cdot \sqrt{21} = 2\sqrt{21}.$

(iii) $\sqrt{12} - \sqrt{75} = \sqrt{4 \cdot 3} - \sqrt{25 \cdot 3} = \sqrt{4} \cdot \sqrt{3} - \sqrt{25} \cdot \sqrt{3} = 2\sqrt{3} - 5\sqrt{3} = -3\sqrt{3}.$

4. Rationalise the denominator of each of the following expressions:

(i) $\frac{2}{\sqrt{6}}$

(ii) $\frac{2}{1-\sqrt{3}}$

(iii) $\frac{x-1}{\sqrt{x}+1}$

Solution

$$(i) \quad \frac{2}{\sqrt{6}} = \frac{2\sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{2\sqrt{6}}{\sqrt{6}^2} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}.$$

$$(ii) \quad \frac{2}{1-\sqrt{3}} = \frac{2 \cdot (1+\sqrt{3})}{(1-\sqrt{3}) \cdot (1+\sqrt{3})} = \frac{2 \cdot (1+\sqrt{3})}{1^2 - (\sqrt{3})^2} = \frac{2 \cdot (1+\sqrt{3})}{1-3} = \frac{2 \cdot (1+\sqrt{3})}{-2} = -(1+\sqrt{3})$$

$$(iii) \quad \frac{x-1}{\sqrt{x}+1} = \frac{(x-1) \cdot (\sqrt{x}-1)}{(\sqrt{x}+1) \cdot (\sqrt{x}-1)} = \frac{(x-1) \cdot (\sqrt{x}-1)}{(\sqrt{x})^2 - 1} = \frac{(x-1) \cdot (\sqrt{x}-1)}{x-1} = \sqrt{x}-1.$$

5. Solve for v the following expression: $a = b + (v-1)\omega, \omega \neq 0$.

Solution

$$a = b + (v-1)\omega \text{ (we use the distributive property)}$$

$$\alpha = \beta + v\omega - \omega \text{ (subtract } b \text{ and add } \omega \text{ from both sides)}$$

$$\alpha - \beta + \omega = v\omega \text{ (we divide both sides by } \omega)$$

$$v = \frac{\alpha - \beta + \omega}{\omega}$$

FOR YOUR OWN PRACTICE

1. Complete the following table:

Number	ex	-5	2	-3	$-\frac{2}{3}$	1,5	0,3333...	$2,\overline{75}$	$\sqrt{2}$	$\sqrt{9}$	π
Natural	x										
Integer	✓										
Rational	✓										
Irrational	x										

2. Complete the following gaps:

A.

$-5 - 7 = \dots\dots\dots$	$\left(-\frac{2}{3}\right) \cdot \left(-\frac{3}{2}\right) = \dots\dots\dots$	$2(x - 3) = \dots\dots\dots$
$3 - 5 = \dots\dots\dots$	$-\frac{10}{-6} = \dots\dots\dots$	$-3 \cdot (2x - 1) = \dots\dots\dots$
$-2 + 2 = \dots\dots\dots$	$\frac{0}{15 - 73} = \dots\dots\dots$	$-5(x + \dots) = \dots\dots + 10$
$-3 \cdot (-5) = \dots\dots\dots$	$\left(-\frac{5}{2}\right) : \left(-\frac{1}{2}\right) = \dots\dots\dots$	$-3 \cdot (\dots - \dots) = 3x - 6$
$-2 \cdot (+3) = \dots\dots\dots$		
$0 \cdot (-5) = \dots\dots\dots$		
$-3 \cdot \frac{2}{5} = \dots\dots\dots$		
$-5 \cdot \frac{1}{5} = \dots\dots\dots$		

B.

$(-2)^2 = \dots\dots\dots$	$(-7)^{-1} = \dots\dots\dots$	$x^m : x^n = \dots\dots\dots$
$-2^2 = \dots\dots\dots$	$\left(-\frac{2}{3}\right)^{-2} = \dots\dots\dots$	$x^{-n} = \dots\dots\dots$
$(-2)^3 = \dots\dots\dots$	$\left(-\frac{1}{2}\right)^{-3} = \dots\dots\dots$	$\left(\frac{x}{y}\right)^0 = \dots\dots\dots$
$-2^3 = \dots\dots\dots$	$\left(-\frac{7}{3}\right)^0 = \dots\dots\dots$	$x^n \cdot y^n = \dots\dots\dots$
$(-2)^{-2} = \dots\dots\dots$		$\frac{1}{x^n} = \dots\dots, \text{ and } \dots\dots$
$(-2)^{-3} = \dots\dots\dots$		$(x^n)^m = \dots\dots\dots$

C.

$3^5 \cdot 3^{-7} = \dots\dots\dots$	$8 \cdot 2^5 = \dots\dots\dots$	$(-2)^7 \cdot 16 = \dots\dots\dots$
$2^6 : 2^{-3} = \dots\dots\dots$	$9 \cdot 2^{-3} = \dots\dots\dots$	$\frac{12^5}{(-3)^5} = \dots\dots\dots$
$(2^{-5})^{-2} = \dots\dots\dots$	$\frac{32}{2^{10}} = \dots\dots\dots$	$8 \cdot 2^{15} \cdot \frac{1}{2^8} = \dots\dots\dots$
$2^5 \cdot 3^5 = \dots\dots\dots$	$4^5 \cdot 7^{10} = \dots\dots\dots$	$\left(\frac{5}{3}\right)^{-10} \cdot \left(-\frac{5}{3}\right)^8 = \dots\dots\dots$
$\frac{12^5}{4^5} = \dots\dots\dots$	$(-3)^2 \cdot 3^5 = \dots\dots\dots$	

D.

$(\sqrt{x})^2 = \dots\dots\dots$	$ x = \begin{cases} \dots\dots, & \text{if } x \geq 0 \\ \dots\dots, & \text{if } x < 0 \end{cases}$
$\sqrt{x^2} = \dots\dots\dots$	$ x = a \Leftrightarrow \dots\dots\dots$
$\sqrt{\alpha \cdot b} = \dots\dots\dots, \text{ if } \dots\dots\dots$	$ x \leq a \Leftrightarrow \dots\dots\dots$
$\sqrt{\alpha^2 \cdot b} = \dots\dots\dots, \text{ if } \dots\dots\dots$	$ x > a \Leftrightarrow \dots\dots\dots$
$\frac{\sqrt{\alpha}}{\sqrt{b}} = \dots\dots\dots, \text{ if } \dots\dots\dots$	$ x + y = 0 \Leftrightarrow \dots\dots\dots$

E.

0.0064579622	
2 d.p =	2 s.f =
4 d.p =	4 s.f =
5 d.p =	5 s.f =
6 d.p =	6 s.f =
7 d.p =	7 s.f =
8 d.p =	8 s.f =

3. Match Column A with Column B. Answers must be written in all CAPITAL letters, and can be selected only once.

A.

Column (A)	Column (B)
1. $(-1)^{2009} \dots\dots$	A. $\frac{1}{9}$
2. $-3^{-2} \dots\dots$	B. 1
3. $9^{-4} \cdot 3^8 \dots\dots$	C. $-\frac{1}{9}$
4. $(3^5 : 3) : 3^6 \dots\dots$	D. -1
	E. 9

B.

Column (A)	Column (B)
1. $(-3)^2$	A. 9
2. -2^3	B. 6
3. -3^2	C. -9
4. $-(-2)^3$	D. -8
	E. 8

C.

Column (A)	Column (B)
1. $(-\alpha + b)^2$	A. $-(\alpha - b)^3$
2. $(-\alpha - b)^3$	B. $-(\alpha - b)^2$
3. $(-\alpha - b)^2$	C. $(\alpha - b)^2$
4. $(-\alpha + b)^3$	D. $(\alpha + b)^2$
	E. $-(\alpha + b)^3$

D.

Column (A)	Column (B)
1. $-\beta^2 - (\alpha + b)(\alpha - b)$	A. α^2
2. $(2x + y^2)^2 - (2y^2 + x)^2$	B. 0
3. $\frac{(4x^2 + 4xy + y^2)}{2x + y} - \frac{(2x + y)^3}{4x^2 + 4xy + y^2}$	C. $\frac{2 + xy}{xy}$
4. $\frac{\left(\frac{1}{x} + \frac{1}{y}\right)^2 - \left(\frac{1}{x} - \frac{1}{y}\right)^2}{2 - xy} - xy$	D. $-\alpha^2$
	E. $3(x^2 - y^4)$

4. Simplify the following expressions:

i) $2\alpha^5 b^3 c^2 4\alpha^2 b^{-3} c^5 \frac{1}{16} \alpha^{-4} b^{-1} c^4$

ii) $\left(\frac{x^2}{y^3}\right)^{-2} \cdot \left(\frac{2y^2}{5x^3}\right)^{-1} \cdot 4xy^{-4}, x, y \neq 0$

5. Rationalise the denominator of each of the following expressions:

i) $\frac{1}{\sqrt{8}} + \frac{1}{\sqrt{2}}$ and ii) $\frac{3}{\sqrt{3}} - \frac{5}{\sqrt{5}}$.

6. Show that $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}) = 1$. Hence, rationalise the denominator of the fraction $\frac{1}{\sqrt{3} - \sqrt{2}}$.

6. Simplify the following expression: $A = \frac{\sqrt{20} - 2\sqrt{8} + 3\sqrt{12}}{\sqrt{45} - 2\sqrt{18} + 3\sqrt{27}}$.

7. Simplify each algebraic fraction:

i) $\frac{x^3 - 4x}{2x^2 - 4x}$

ii) $\frac{x^3 - 2x^2 + x}{x^2 - x}$

iii) $\frac{(x^2 - x) + 2x - 2}{x^2 - 1}$

iv) $\frac{x(x-2)+1}{(x-2)(x-1)}$

v) $\frac{x^2 + x + 1}{x+1} \cdot \frac{x^2 - 1}{x^3 - 1}$

vi) $\left(x - \frac{1}{x}\right)^2 \cdot \frac{x^3 + x^2}{(x+1)^3}$

vii) $\frac{x^2 - 3x + 2}{x^2 - x} \cdot \frac{x^2 + 2x}{x^2 + x - 2}$

viii) $\frac{\alpha^2 + \beta^2 + 2\alpha\beta}{3(\alpha + \beta)}$

ix) $\frac{\alpha x + \alpha y + \beta x + \beta y}{x^2 - y^2}$

x) $\frac{36x^2 - 12x + 1}{48x - 8}$

xi) $\frac{x^2 - 9y^2}{2x^2 - 12xy + 18y^2}$

xii) $\frac{164\alpha^2 b^3 - 8\alpha^3 b^3}{4\alpha^2 b^2}$

xiii) $\frac{(x+h)^2 - x^2}{h}$

xiv) $\frac{x^2 - 1}{(1-x)^2}$

xv) $\frac{x^2 - (\alpha + \beta)^2}{\alpha^2 - (x + \beta)^2}$

8. Write in the standard form the following numbers:

i) 30.000.000

ii) 5600000

iii) 243000000

iv) 0,00002

v) 0,00000034

vi) 0,000000000735

9. Solve for....

i) $xy = z$ (Solve for x)

ii) $a = xy$ (Solve for y)

iii) $\frac{a}{b} = z$ (Solve for b)

iv) $\frac{x}{y} = \frac{a}{k}$ (Solve for k)

v) $a + bx = c$ (Solve for b)

vi) $a = \frac{x-y}{b}$ (Solve for x)

vii) $x = \frac{k}{a-b}$ (Solve for a)

viii) $a = 1 - \frac{x}{y}, a \neq 0$ (Solve for y)

10. In June Demar worked a total of 45.5 hours, in July he worked 35 hours – by what percentage did Dylan's working hours decrease in July?

