SECTION: EXPONENTS and LOGARITHMS

UNITS:

1. Exponents.
2. Laws of Exponents.
3. Rational Exponents.
4. Exponential Equations.
5. Logarithms.
6. Solving equations involving logarithms.

WHAT YOU NEED TO KNOW

1. Exponents

- If **n** is a **positive integer**, then **α^n** is the product of **n** factors of **α**.

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}$$

We say that **α** is the **base, and** n **is the exponent** or **index**.

Also we define: $a^0 = 1$ and $a^1 = a$.

- If **n** is a **negative integer**, then

$$a^{-n} = \frac{1}{a^n} = \frac{1}{\underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}}$$

Also, we define: $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$.

ex.1: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ ex.2: $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^{2} = \frac{4}{3} \cdot \frac{4}{3} = \frac{16}{9}$.

2. Laws of Exponents

If $a, b > 0$ and $x, y \in R$, then:

1. $a^x \cdot a^y = a^{x+y}$

2. $\dfrac{a^x}{a^y} = a^{x-y}$

3. $(a \cdot b)^x = a^x \cdot b^x$

4. $(a^x)^y = a^{x \cdot y}$

5. $(\dfrac{a}{b})^x = \dfrac{a^x}{b^x}$

6. $a^{-x} = \dfrac{1}{a^x}$

7. $\left(\dfrac{a}{b}\right)^{-x} = \left(\dfrac{b}{a}\right)^x$

8. Don't forget that $\begin{cases} (-\alpha)^n = \alpha^n, & \text{where n is even} \\ (-\alpha)^n = -\alpha^n, & \text{where n is odd} \end{cases}$

i.e A negative base raised to an even number exponent is a positive number but a negative base raised to an odd number exponent is a negative number.

ex:

$(-2)^2 = +4 \quad \text{but} \quad -2^2 = -4$

$(-2)^3 = -8 \quad \text{but} \quad -2^3 = -8$

3. Rational Exponents

- The exponent laws used previously can be applied to rational exponents (**i.e** exponents which are written as a fraction)

- We define:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}, \text{ for } a>0,\ n\in\mathbb{Z}^+,\ m\in\mathbb{Z}$$ and read "*The nth root of α raised to m*".

ex.1: $4^{\frac{1}{2}} = \sqrt[2]{4} = 2$ ex.2: $\sqrt[3]{2^6} = 4^{\frac{6}{3}} = 4^2 = 16$.

- Also, using the above exponents laws we get:

$$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{\sqrt[n]{a^m}}, \text{ for } a>0,\ n\in\mathbb{Z}^+,\ m\in\mathbb{Z}.$$

ex.1: $8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{8^2}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$ ex.2: $\frac{1}{\sqrt{2}} = \frac{1}{2^{\frac{1}{2}}} = 2^{-\frac{1}{2}}$.

4. Exponential Equations

- An **exponential equation** is an equation in which the unknown occurs as part of the exponent.

ex1: $2^x = 8$ or ex2: $3^{x-1} = 27$

- To **solve the above equations** (*algebraically*), we have to create same base numbers from both sides of equation in order to equate the exponents.

ex1:
$$2^x = 8$$
$$2^x = 2^3$$
$$x = 3$$

or ex2:
$$3^{x-1} = 27$$
$$3^{x-1} = 3^3$$
$$x-1 = 3$$
$$x = 4$$

Notice that: The exponential equation $\alpha^x = b,\ b\le 0$ is an impossible equation. ex. $2^x = -8$.

WORKED EXAMPLES

1. Solve the following equations

i) $2^x = 32$.

ii) $3^x = -27$.

iii) $5^{x^2-5x+6} = 1$.

iv) $2^{2x+1} = 4 \cdot 2^x$.

v) $9^x - 2 \cdot 3^x - 3 = 0$.

Solution

In order to solve this equation, we have to create the same base at both sides of the equation. So we have:

i) $2^x = 32 \Leftrightarrow 2^x = 2^5 \Leftrightarrow x = 5$.

ii) $3^x = -27 < 0$ impossible.

iii) $5^{x^2-5x+6} = 1 \Leftrightarrow 5^{x^2-5x+6} = 5^0 \Leftrightarrow x^2 - 5x + 6 = 0 \Rightarrow \begin{Bmatrix} x_1 = 2 \\ x_2 = 3 \end{Bmatrix}$.

iv) $2^{2x+1} = 4 \cdot 2^x \Leftrightarrow 2^{2x+1} = 2^2 \cdot 2^x \Leftrightarrow 2^{2x+1} = 2^{2+x} \Leftrightarrow 2x + 1 = 2 + x \Leftrightarrow x = 1$.

v) $9^x - 2 \cdot 3^x - 3 = 0 \Leftrightarrow \left(3^2\right)^x - 2 \cdot 3^x - 3 = 0 \Leftrightarrow \left(3^x\right)^2 - 2 \cdot 3^x - 3 = 0 \quad (1)$

Let $3^x = y \overset{(1)}{\overbrace{\Rightarrow}} y^2 - 2y - 3 = 0 \Rightarrow \begin{Bmatrix} y_1 = 3 \\ y_1 = -1 \end{Bmatrix}$.

Finally, $3^x = 3 \Leftrightarrow x = 1$ or $3^x = -1 \Leftrightarrow rejected$

2. Solve the following simultaneous equations: $\begin{cases} 9^{x+1} = 3^{y+3} & (1) \\ 4^{x+y} = 8 \cdot 2^x & (2) \end{cases}$.

Solution

In order to solve this equation, we have to create the same base at both sides of the equation. So we have:

$(1) \Rightarrow 9^{x+1} = 3^{y+3} \Leftrightarrow \left(3^2\right)^{x+1} = 3^{y+3} \Leftrightarrow 3^{2(x+1)} = 3^{y+3} \Leftrightarrow 2x+2 = y+3 \Leftrightarrow 2x-y=1$.

$(2) \Rightarrow 4^{x+y} = 8\cdot 2^x \Leftrightarrow \left(2^2\right)^{x+y} = 2^3\cdot 2^x \Leftrightarrow 2^{2(x+y)} = 2^{3+x} \Leftrightarrow 2x+2y = 3+x \Leftrightarrow x+2y=3$

So, we have to solve the following simultaneous equations: $\begin{cases} 2x-y=1 \\ x+2y=3 \end{cases}$.

Using the substitution method, we get:

$$\begin{Bmatrix} 2x-y=1 \\ x+2y=3 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} y=2x-1 \\ x+2y=3 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} y=2x-1 \\ x+2\cdot(2x-1)=3 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} y=2x-1 \\ x+4x-2=3 \end{Bmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{Bmatrix} y=2x-1 \\ 5x=5 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} y=2x-1 \\ x=1 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} y=2\cdot(1)-1 \\ x=1 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} y=1 \\ x=1 \end{Bmatrix}$$

3. Solve the following equation $3^x + 3^{x-1} = \frac{45}{3^{x+2}} + \frac{7}{3^x}$.

Solution

In order to solve this equation, we have to create the same base at both sides of the equation. So we have:

$$3^x + \frac{3^x}{3} = \frac{45}{3^x\cdot 3^2} + \frac{7}{3^x} \Leftrightarrow 3^x\left(1+\frac{1}{3}\right) = \frac{45}{3^x\cdot 9} + \frac{7}{3^x} \Leftrightarrow 3^x\,\frac{4}{3} = \frac{5}{3^x} + \frac{7}{3^x} \Leftrightarrow$$

$$\Leftrightarrow 3^x\,\frac{4}{3} = \frac{12}{3^x} \Leftrightarrow 4\cdot(3^x)^2 = 3\cdot 12 \Leftrightarrow (3^x)^2 = \frac{36}{4} \Leftrightarrow 3^{2x} = 9 \Leftrightarrow 3^{2x} = 3^2 \Leftrightarrow$$

$$\Leftrightarrow 2x = 2 \Leftrightarrow x = 1$$

4. Solve the following equation $3^{2x} + 9^x = 11\cdot 4^{x-1} + 4^{x+1}$.

Solution

$$9^x + 9^x = 11\cdot\frac{4^x}{4} + 4^x\cdot 4 \Leftrightarrow 2\cdot 9^x = 4^x\left(\frac{11}{4}+4\right) \Leftrightarrow 2\cdot 9^x = 4^x\left(\frac{27}{4}\right)$$

$$\Leftrightarrow 8\cdot 9^x = 27\cdot 4^x \Leftrightarrow \frac{9^x}{4^x} = \frac{27}{8} \Leftrightarrow \frac{3^{2x}}{2^{2x}} = \frac{3^3}{2^3} \Leftrightarrow (\frac{3}{2})^{2x} = (\frac{3}{2})^3 \Leftrightarrow$$

$$\Leftrightarrow 2x = 3 \Leftrightarrow x = \frac{3}{2}$$

5. Solve the following equation $4^{x-1} - 5\sqrt{4^{x-2}} + 1 = 0$ (1).

Solution

$$(1) \Rightarrow \frac{4^x}{4} - 5\cdot\sqrt{\frac{4^x}{4^2}} + 1 = 0 \Leftrightarrow \frac{1}{4}\cdot 4^x - 5\cdot\frac{\sqrt{4^x}}{4} + 1 = 0 \Leftrightarrow 4^x - 5\cdot\sqrt{4^x} + 4 = 0 \text{ (2).}$$

Let $\sqrt{4^x} = y$ (3), so : $(2) \Rightarrow y^2 - 5y + 4 = 0$.

$$\Delta = 9, y_{1,2} = \frac{5\pm 3}{2} \Leftrightarrow \begin{cases} y_1 = 4 \\ y_2 = 1 \end{cases}$$

Finally, from the equation (3) we get: $\begin{cases} \sqrt{4^x} = 4 \Leftrightarrow 4^x = 16 \Leftrightarrow 4^x = 4^2 \Leftrightarrow x = 2 \\ \sqrt{4^x} = 1 \Leftrightarrow 4^x = 1 \Leftrightarrow x = 0 \end{cases}$

6. A population of bacteria grows according to the model $p(t) = p_0 \cdot 2^{k\cdot t}, k > 0$, when p(t) is the size of the population after t hours. Given that after 2 hours there are 400 bacteria and after 4 hours 3.200 bacteria, find

a) The value of k

b) The initial size of bacteria

c) The size of population after 6 hours

d) By what percentage is the population of the bacteria increase at the first hour compared with the initial amount?

e) The time where the amount of bacteria is going to be double from the initial.

Solution

a) Given that after 2 hours there are 400 bacteria we get that:
$p(2) = p_0 \cdot 2^{k\cdot 2} \Rightarrow 400 = p_0 \cdot 2^{k\cdot 2}\,(1)$.

Given that after 4 hours there are 3200 bacteria we get that:
$p(4)=p_0\cdot 2^{k\cdot 4}\Rightarrow 3200=p_0\cdot 2^{k\cdot 4}\,(2)$.

$$\frac{(2)}{(1)}\Rightarrow\frac{3200}{400}=\frac{p_0\cdot 2^{k\cdot 4}}{p_0\cdot 2^{k\cdot 2}}\Leftrightarrow 8=2^{2k}\Leftrightarrow 2^3=2^{2k}\Leftrightarrow 3=2k\Leftrightarrow k=\frac{3}{2}.$$

b) For k=3/2 we get:

$$(1)\Rightarrow 400=p_0\cdot 2^{2\cdot\frac{3}{2}}\Leftrightarrow 400=p_0\cdot 2^3\Leftrightarrow 400=p_0\cdot 2^3\Leftrightarrow 400=p_0\cdot 8\Leftrightarrow p_0=50.$$

c) $$\frac{p(1)-p(0)}{p(0)}\cdot 100\%=\frac{50\cdot 2^{\frac{3}{2}}-50}{50}\cdot 100\%=182{,}84\%.$$

d) For t = 6 we get: $p(6)=50\cdot 2^{\frac{3}{2}\cdot 6}=50\cdot 2^9=50\cdot 512=25.600$ bacteria.

e) For P(t) = 2Po we have: $2p_o=p_o\cdot 2^{\frac{3}{2}\cdot t}\Leftrightarrow 2=2^{\frac{3}{2}\cdot t}\Leftrightarrow 1=\frac{3}{2}\cdot t\Leftrightarrow t=\frac{2}{3}h$.

FOR YOUR OWN PRACTICE

1. Simplify the followings:

$\text{i)}\,2^4 2^3\quad \text{ii)}\,2^4(-2)^3\quad \text{iii)}\,(5^{-2})(5^{-4})^{-2}\quad \text{iv)}\,(-1)^4(-2)^{-2}4^2\quad \text{v)}\,(-16)^3:(4^{-2})^{-3}$

$\text{vi)}\,(-9)^{-2}:9^{-2}\quad \text{vii)}\,2^{-5}:(2^3:2^6)\quad \text{viii)}\,(3^{-4}:3^4):3^{-5}$

$\text{ix)}\,(-3)^2+3^2+(-1)^9+(-1^{10})\quad \text{x)}\,4^{-2}-2^{-3}+[(-2)^3]^{-2}$

2. Simplify the followings:

$\text{i)}\,-(2x^3)^{-2}\quad \text{ii)}\left(-\frac{2x^2}{3}\right)^3\quad \text{iii)}\,\frac{6x^7}{3x^3}\quad \text{iv)}\left(\frac{-1}{2x}\right)^{-2}\quad \text{v)}\,\frac{-9x^2y}{3xy^2}$

$\text{vi)}\left(\frac{1}{2}x^4y^3\right)\left(\frac{1}{3}x^4y^{-5}\right)\quad \text{vii)}\left(\frac{1}{3}x^{-6}y\right)(x^4y^{-1})\quad \text{viii)}\left(\frac{-3x^4y^{-4}}{2x^{-1}y^{-1}}\right)^2$

$\text{ix)}\left(\frac{-2x^3y^{-5}}{6x^{-2}y^{-2}}\right)^3\quad \text{x)}\left(\frac{-x^2y^{-3}}{x^{-1}y^{-4}}\right)^{-2}\quad \text{xi)}\left(\frac{-4x^8y^{-2}}{8x^{-1}y^{-5}}\right)^{-3}$

$\text{xii)}\,(x^4y^5)^{-2}(x^{10}y^7)^0(x^2y^1)^4(x^2)^3(y^2)^4(x^{-3})^2(y^{-2})^{-1}$

$\text{xiii)}\left(\frac{x^3}{y^2}\right)^{-2}\left(\frac{-2y^3}{x^4}\right)^{-3}\quad \text{xv)}\left(\frac{1}{8}x^{-1}y^4\right)\left(\frac{x^2}{y^2}\right)^{-3}\left(\frac{-2x^2}{y^3}\right)^{-4}$

3. Solve the following equations:

- **i)** $2^x = 64$
- **ii)** $(\frac{1}{2})^x = \frac{1}{8}$
- **iii)** $(\frac{1}{2})^x = 4$
- **iv)** $3^{-x} = \frac{1}{81}$
- **v)** $(\frac{3}{4})^x = \frac{64}{27}$
- **vi)** $27^{4x} = 9^{x+1}$
- **vii)** $32^x = 16^{1-x}$
- **viii)** $3^{x^2-x-2} = 1$
- **ix)** $2^{2x+1} - 4\cdot 2^x = 0$
- **x)** $2\cdot 4^x - 5\cdot 2^x + 2 = 0$
- **xi)** $3^{2x+1} - 26\cdot 3^x - 9 = 0$.
- **xii)** $21\cdot 3^x + 5^{x+3} = 3^{x+4} + 5^{x+2}$.
- **xiii)** $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$.
- **xiv)** $3^{2x} = \frac{1}{81}$.
- **xv)** $(\frac{1}{3})^x = 27$.
- **xvi)** $2^{-x} = 32$.
- **xvii)** $(\frac{1}{3})^{-x} = 27$.
- **xviii)** $\frac{1}{2^x} = 16$.
- **xix)** $2^{x^2-5x+6} = 1$.
- **xx)** $[3^{(x^2-9)}]^{(x-2)} = 1$.
- **xxi)** $4^{3x} = 2^4\cdot 16^{\frac{x}{2}}$.
- **xxii)** $9^x - 2\cdot 3^x - 3 = 0$.
- **xxiii)** $3^{2x-2} + 3^x = 4$.
- **xxiv)** $5^{2x-1} + 5^{x+1} = 250$.
- **xxv)** $2^x - 5\sqrt{2^x} + 4 = 0$.
- **xxvi)** $5\cdot 2^x = 2^{x+3} - 3\sqrt{2}$.
- **xxvii)** $3^{x+1} - 28 + 9\cdot 3^{-x} = 0$.
- **xxviii)** $2^{x-2} - 3^{x-3} - 2^{x-3} + 3^{x-4} = 0$.
- **xxix)** $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$.
- **xxx)** $(x^2 - 5x + 5)^{x+2} = 1$.
- **xxxi)** $e^{2x} + e = e^x + e^{x+1}$.

4. Solve the following simultaneous equations:

i) $\begin{cases} 8^{2x+1} = 32 \cdot 4^{4y-1} \\ 5 \cdot 5^{x-y} = 5^{2y+1} \end{cases}$.

ii) $\begin{cases} 3^x + 2^y = 11 \\ 3^x - 2^y = 7 \end{cases}$.

iii) $\begin{cases} \dfrac{e^x}{e^y} = 1 \\ e^x \cdot e^y = e^2 \end{cases}$.

iv) $\begin{cases} 2^x \cdot 2^y = 8 \\ 2^x + 2^y = 6 \end{cases}$.

v) $\begin{cases} 3^y - 2^x = 1 \\ 3^y + 16 \cdot 2^{-x} = 11 \end{cases}$.

vi) $\begin{cases} 2^x \cdot 5^y = 250 \\ 2^y \cdot 5^x = 40 \end{cases}$.

vii) $\begin{cases} 9^{x+1} = 3^{y+3} \\ 4^{x+y} = 8 \cdot 2^x \end{cases}$.

viii) $\begin{cases} 2^{x^2-5x+6} = 1 \\ x + y = 8 \end{cases}$.

ix) $\begin{cases} 2^{x-1} \cdot 4^y = 1 \\ 3^x \cdot 3^{y-1} = 9 \end{cases}$.

x) $\begin{cases} 3^x - 5^y = 4 \\ 9 \cdot 3^{-x} + 5^y = 6 \end{cases}$.

5. A high-fever patient is given an anti-fever pill. The temperature θ (t), t hours after taking the drug, is given by $\theta(t) = 36 + 4 \cdot (\frac{1}{2})^t$ (in Celsius).

- **a.** What is the temperature of the patient when the drug was given to him?
- **b.** After how many hours the patient's temperature will get to the normal value of 36.5^0 C;
- **c.** If the drug lasts for 4 hours, what will be the patient's temperature when the effect of the drug expires?

5. Logarithms

- **The relationship between exponents and logarithms**

If $a>0, a\neq 1$ and $k>0$, then: $a^x = k \Leftrightarrow x=\log_a k$.

Examples

1. $2^x = 5 \Leftrightarrow x=\log_2 5$.
2. $2^x = 16 \Leftrightarrow x=\log_2 16$ so $\log_2 16 = 4$, since $2^4=16$.
3. $3^x = \frac{1}{9} \Leftrightarrow x=\log_3 \frac{1}{9}$ so $\log_3 \frac{1}{9} = -2$, since $3^{-2}=\frac{1}{9}$.
4. $5^{-x} = 125 \Leftrightarrow -x=\log_5 125 \Leftrightarrow x=-\log_5 125$ so $\log_5 125 = 3$, since $5^3=125$.

- **Two common abbreviations for logarithms to particular bases**

1. $10^x = k \Leftrightarrow x=\log_{10} k$. The $\log_{10} k$ often written as $\log k$ (**Common Logarithm**).
2. $e^x = k \Leftrightarrow x=\log_e k$. The $\log_e k$ often written as $\ln k$ (**Natural Logarithm**).

6. Solving equations involving logarithms

- We can solve an equation involving logarithms either using the definition of logarithm (**ex 1, 2**) either creating logarithms with the same base numbers from both sides of equation (**ex 3**).

ex1. $2^x = 5 \overbrace{\Leftrightarrow}^{definition} x=\log_2 5 = 2{,}3219...$

ex2. $\log_3(x-1)=2 \overbrace{\Leftrightarrow}^{definition} x-1=3^2 \Leftrightarrow x=8+1 \Leftrightarrow x=9$.

ex3. $\log_2(2x+1)=\log_2 3 \Leftrightarrow 2x+1=3 \Leftrightarrow 2x=2 \Leftrightarrow x=1$.

WORKED EXAMPLES

1. Solve the following equation $5^x = 3^{1-x}$.

Solution

$5^x = 3^{1-x} \Leftrightarrow 5^x = \dfrac{3}{3^x} \Leftrightarrow 5^x \cdot 3^x = 3 \Leftrightarrow 15^x = 3$ and by using the definition we have $15^x = 3 \Leftrightarrow x = \log_{15} 3$.

2. Solve the following equation $\log(\frac{x^2+1}{x}) = \log 2$.

Solution

$$\log\frac{x^2+1}{x} = \log 2 \Leftrightarrow \frac{x^2+1}{x} = 2 \Leftrightarrow$$

$$\Leftrightarrow x^2+1 = 2x \Leftrightarrow x^2 - 2x + 1 = 0 \Leftrightarrow (x-1)^2 = 0 \Leftrightarrow x-1 = 0 \Leftrightarrow x = 1$$

3. Solve the following system of equations $\begin{cases} xy = 8 \\ \log y = \log x^2 \end{cases}$.

Solution

$$\begin{cases} xy = 8 \\ \log y = \log x^2 \end{cases} \Leftrightarrow \begin{cases} xy = 8 \\ y = x^2 \end{cases} \Leftrightarrow \begin{cases} y = \dfrac{8}{x} \\ \dfrac{8}{x} = x^2 \end{cases} \Leftrightarrow \begin{cases} y = \dfrac{8}{x} \\ x^3 = 8 \end{cases} \Leftrightarrow \begin{cases} y = 4 \\ x = 2 \end{cases}$$

FOR YOUR OWN PRACTICE

1. Complete the following gaps:

i) $\ln e^2 = ...$
ii) $\log_{....} 25 = 2$
iii) $\log_{\frac{1}{3}} .... = -2$
iv) $\log_\alpha \sqrt{\alpha} = ...,\alpha > 1$
v) $\log_7 \frac{1}{7} = ...$
vi) $\ln .... = -3$
vii) $\log_\alpha .... = 2, \alpha > 1$
viii) $\log_{.....} \alpha^2 = 1$.

2. Find the following logarithms:

i) $\log 0,001$
ii) $\log_{\frac{1}{10}} \sqrt{10}$
iii) $\log_{\frac{1}{2}} 32$
iv) $\log_9 \frac{\sqrt{27}}{3}$
v) $\log_{\sqrt{2}} 16$
vi) $\log_{\frac{3}{2}} \sqrt{\frac{8}{27}}$

3. Find x in following equations:

i) $\log x = 3$
ii) $\log_4 x = -\frac{1}{2}$
iii) $\log_{\sqrt{2}} x = \frac{2}{3}$
iv) $\log_x 16 = 4$
v) $\log_x 8 = \frac{3}{2}$
vi) $\log_x 0,1 = -3$.

4. Solve the following equations:

i) $2^{2x-1}=5$
ii) $5^x=2^{1-x}$
iii) $3^{x-1}=2^{x+1}$
iv) $\log\sqrt{x}=1$
v) $\ln\left(x^2-1\right)=0$
vi) $\log(1+x)=\log(1-x)$
vii) $\ln\left(\frac{x}{2}\right)^2=\ln x$
viii) $\log(1+x)=\log\left[10\cdot(1-x)\right]$
ix) $\log\left[(x+1)\cdot(x-1)\right]=\log 2$

5. Under constant temperature, the atmospheric pressure p at a height h is given by $p=101300\cdot e^{k\cdot h}$.
i) Find the value of k if at the height of 3050m the pressure is 68900 Pa.
ii) What is the pressure at the height of 1000m?

6. In a lake it is found that the intensity of light decreases as it passes through water. The intensity *I(x)* units at depth x metres from the surface is given by $I(x)=I_0\cdot 10^{-0,5x}, x\geq 0$, where I_0 units is the intensity at the surface. Find the depth at which the intensity would be $0,1\cdot I_0$.

7. The number of bacteria in a culture, after t hours, is given by $Q(t)=Q_0\cdot e^{0,34t}$, where Q_0 the initial number of bacteria. How long will it take for the size of the population to double?

8. The sales s(t) (in thousand units) of a product, t years after first introduced to the market is given by $s(t)=100(1+e^{kt})$.
i) Find the value of k if the sales at the end of the first year are 15000 units.
ii) How many units will be sold during the first 5 years?